In reality, when deposition occurs there exists a sticking probability which is related to the adhesion of particles at the immediate vicinity of the wall, even though there is no drifting motion due to field effects [8]. The adhesive forces are either electrical or liquid (viscosity and surface tension) in nature. The observed mean deposition velocity therefore includes the probability that some droplets do not merge into the wall layer, but rebound and are removed by turbulent diffusion. There is, however, the possibility that the rivulent flow in a steel tube might be disturbed differently in an acrylic tube, even though the two flow rates were the same. Since sticking on a rivulet might be different from sticking on a dry surface, the overall deposition rate in the rivulet flow of an acrylic tube might be different from that in a steel tube. Because the entrainment in the present case is insignificant, and also due to the agreement of the deposition data for the two test surfaces, it can be inferred that the sticking probability for these conditions is close to unity. It therefore appears that the deposition rates for dry wall dispersed flow, rivulet dispersed flow, and annular dispersed flow are all represented by the data of Fig. 2, for the conditions studied here.

It can be concluded from the above results and discussion that for the conditions studied in the present work representing drop sizes and flow Reynolds number typical of many two-phase flow applications, the data for adiabatic acrylic tube may be used directly in the heat transfer analysis of dispersed two-phase flow in metallic tubes provided that there is no significant droplet entrainment. It should, however, be recognized that in some cases the condition of adiabatic tube may be much different from the condition of a heated tube due to non-uniform evaporation of droplet, vapor generation near the tube surface and change of surface tension at high temperatures, etc. Therefore, in such situations, the conclusion of this experiment may not be able to be applied to heated tubes directly without justification.

Acknowledgement—This work was performed with support of the National Science Foundation under grants ENG78-06211 and ENG80-05681.

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Int. J. Heat Mass Transfer. Vol. 25, No. 3, pp. 424–425, 1982
Printed in Great Britain

0017-9310/82/030424-02 $03.00/0
Pergamon Press Ltd.

MINIMUM MASS CONVECTIVE ANNULAR FIN

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(*Received* 16 *April* 1981)

INTRODUCTION

IN A RECENT paper Mikk [1] has considered the problem of minimizing the mass of an annular fin on a cylindrical base of specified radius that rejects heat to the surroundings by convection at a specified rate. He asserts that, contrary to the results of Schmidt [2] and Duffin [3], the temperature at the tip of the minimum mass fin is not the same as that of the ambient fluid. In this paper we call attention to a logical flaw in his analysis that invalidates his conclusions. Moreover, if this flaw is corrected, his analysis then leads immediately to the earlier results.

We will use Mikk's notation and refer to equations in his paper without further explanation.

NOMENCLATURE

- l, fin height [m];
- q, dimensionless heat flux Q/Q_1;
- Q, heat flux per meter of fin base [W/m];
- r, radius [m];
- v, dimensionless volume $2\lambda\alpha^2\vartheta_1^3 V/Q_1^3$;
- V, fin volume per meter of fin base [m²];
- α, heat transfer coefficient [W/(m²K)];
- δ, fin thickness [m];
- ε, dimensionless parameter $Q_1/2\alpha r_1\vartheta_1$;
- ϑ, excess of fin temperature over temperature of ambient fluid [K];
- θ, dimensionless temperature ϑ/ϑ_1;
- λ, thermal conductivity [W/(m·K)];
- ρ, dimensionless radius r/l;
- σ, dimensionless parameter $\alpha l^2/\lambda\delta_1$;
- φ, radius ratio r_2/r_1;
- χ, dimensionless parameter $Q_1 l/\lambda\vartheta_1\delta_1$;
- Δ, dimensionless fin thickness δ/δ_1.

Subscripts

- 1, fin base;
- 2, fin tip.

THE FLAW IN MIKK'S ANALYSIS

The difficulty arises in the transition from Mikk's equations (29) and (30) to (31). Mikk assumes in this derivation that θ_2 and φ are independent variables, so that φ can be held constant while differentiating with respect to θ_2. It is obvious, however, from his equation (1) that $l = r_1(\varphi - 1)$, and then from his equations (2) and (4) that $\sigma/\chi = \alpha l\vartheta_1/Q_1 = (\varphi - 1)/2\varepsilon$ if ε is defined as $Q_1/2\alpha r_1\vartheta_1$. Moreover, it follows from his equations (19) and (23) that $\sigma/\chi = 3/[\varphi + 2 + (2\varphi + 1)\theta_2]$. We conclude that

$$\theta_2 = \frac{6\varepsilon - (\varphi - 1)(\varphi + 2)}{(2\varphi + 1)(\varphi - 1)}. \quad (1)$$

For fixed fluid conditions and fixed surface properties of the fin, the quantities α and ϑ_1 are constant. The cylinder radius r_1 and the heat transfer rate Q_1 (or $2\pi r_1 Q_1$) are also specified. Therefore, ε is constant and θ_2 and φ are not independent variables in Mikk's equation (29).

If due account is taken of our equation (1), we can correctly determine that

$$\frac{\partial v}{\partial \theta_2} = -\frac{(2\varphi+1)^2\varphi(\varphi-1)^4[\varphi^2+\varphi-2(1+3\varepsilon)]^2}{54\varepsilon^3[\varphi^2-2\varphi(1+4\varepsilon)+1+2\varepsilon](\varphi^2-1-2\varepsilon)^2}$$

$$= -\frac{(2\varphi+1)^4\varphi(\varphi-1)^6\theta_2^2}{54\varepsilon^3[\varphi^2-2\varphi(1+4\varepsilon)+1+2\varepsilon](\varphi^2-1-2\varepsilon)^2}. \quad (2)$$

We observe that, if $\theta_2 < 0$, then Mikk's equation (9) would imply that $dq/d\rho > 0$ when $\rho = \rho_2$, his equation (13) would then imply that $q < 0$ if ρ is close to (but not equal to) ρ_2, his equation (20) would imply that $d\theta/d\rho < 0$ for all ρ, and finally his equation (8) would imply that $\Delta < 0$ if ρ is close to (but not equal to) ρ_2. Although Mikk nowhere mentions it, this last implication is clearly not acceptable, i.e. the only fin profiles that make sense are such that $\Delta > 0$ when $\rho_1 \le \rho < \rho_2$. Moreover, it follows from Mikk's equation (9) that $\theta_2 < 1$. Therefore, the only values of θ_2 that make sense lie on the half-open interval $[0, 1)$.

Because $\varphi > 1$ (obviously) for any finite r_1, we infer from (1) that $\varphi^2 + \varphi \le 2(1 + 3\varepsilon)$; hence the factor $\varphi^2 - 2\varphi(1 + 4\varepsilon) + 1 + 2\varepsilon$ in the denominator of (2) does not exceed $(3 + 8\varepsilon)(1 - \varphi)$, and so is negative. In addition, it follows from (1), or the argument leading to (1), that

$$\frac{\varphi^2 - 1}{2\varepsilon} = \frac{3(\varphi + 1)}{\varphi + 2 + (2\varphi + 1)\theta_2} > \frac{3(\varphi + 1)}{\varphi + 2 + 2\varphi + 1} = 1$$

because $\theta_2 < 1$. We thus see that the factor $\varphi^2 - 1 - 2\varepsilon$ in the denominator of (2) is positive. We conclude from (2) that $dv/d\theta_2 > 0$ unless $\theta_2 = 0$. The minimum value of v therefore occurs when $\theta_2 = 0$, i.e. when $\varphi 2 + \varphi = 2(1 + 3\varepsilon)$, and is such that

$$v_{\min} = 9(\varphi + 1)/(\varphi + 2)^3$$

$$= (9/16\varepsilon^3)\{1 + 4\varepsilon + (8/3)\varepsilon^2 - [1 + (8\varepsilon/3)]^{3/2}\}. \quad (3)$$

Our equation (3) is of course the result that would have been obtained by Schmidt [2] or Duffin [3] if they had carried the details of their analyses to their ultimate conclusion. The numerical results described in Mikk's Table 1 as 'Schmidt's data' are compatible with our equation (3).

The numerical comparisons made by Mikk in his Table 1 should be interpreted as follows. Consider, for example, the case of $\varphi = 3$. From his equations (31) and (29) we see that $\theta_2 = 1/3$, $v = 243/968 = 0.25103$ (this value agrees with his Table 1). But we see from our equation (1) that $\varepsilon = 22/9$ when $\varphi = 3, \theta_2 = 1/3$, and with this value of ε the minimum value of v is calculated from our equation (3) as 0.23477, a value less than 0.25103, as it should be. Note that when $\varepsilon = 22/9$ the values of φ and θ_2 for the optimum fin profile are $(609^{1/2} - 3)/6 = 3.6130$ and 0, respectively.

Mikk's analysis can be reinterpreted as furnishing the correct answer to a different mathematical problem than the one posed at the begining of this paper. He has found the minimum mass of an annular fin on a cylindrical base that rejects heat to the surroundings by convection at a specified rate (measured in W/m of fin base) when the radius r_1 of the cylinder is not specified in advance, but is free to vary subject to the constraint that $\varphi = r_2/r_1$ is fixed. We do not believe that this mathematical problem has any useful engineering application.

If r_1 is free to vary without any restriction, then ε is arbitary in equation (3). It is not sufficient now to minimize v, because the actual volume is $2\pi r_1 V = (\pi r_1 Q_1^3/\lambda\alpha^2\vartheta_1^3)v = (\pi Q_1^4/2\lambda\alpha^3\vartheta_1^4)v\varepsilon^{-1}$ if Q_1 is fixed, or $\{(2\pi r_1 Q_1)^2/2\pi\lambda\alpha_1\}v\varepsilon$ if $2\pi r_1 Q_1$ is fixed. It now matters whether the heat rejected per meter of fin base, Q_1, or the total heat rejected, $2\pi r_1 Q_1$, is fixed, because

$$\frac{d(v\varepsilon^{-1})}{d\varepsilon} = \frac{-4(1+3\varepsilon)}{3\varepsilon^2[1+3\varepsilon+(4/3)\varepsilon^2+\{1+(20\varepsilon/3)\}\{1+8\varepsilon/3)\}^{1/2}},$$

$$\frac{d(v\varepsilon)}{d\varepsilon} = \frac{4}{3[1+2\varepsilon+\{(1+(2\varepsilon/3)\}\{1+(8\varepsilon/3)\}^{1/2}]}.$$

If Q_1 is fixed, the minimum volume occurs when $\varepsilon = \infty$, i.e. when $r_1 = 0$, and is zero. If $2\pi r_1 Q_1$ is fixed, the minimum volume occurs when $\varepsilon = 0$, i.e. when $r_1 = \infty$, and is also zero. Of course $Q_1 = 0$ in the latter case, and neither of these limiting solutions is very interesting.

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